SOLUTION OF A MINIMUM COST NETWORK FLOW PROBLEM IN A MULTI-WAREHOUSE MULTI-RETAILER DISTRIBUTION SYSTEM WITH GENETIC ALGORITHM

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ABSTRACT

The delivery of goods from suppliers to local customers at minimum costs is an important problem in logistics distribution system. In this paper, a multi-warehouse multi-retailer distribution system, considered from the real life, has been modeled as a network flow problem. Genetic Algorithm (GA) which is a heuristic optimization technique has been used to find the optimum solution (minimum cost) for the model. In the end, the results are reported and compared with the results which are calculated with Lingo 4.0.

Keywords: Distribution system, Transportation problem, Optimization, Network Flows, Genetic Algorithm.
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1. INTRODUCTION

Logistics is often defined as the art of bringing the right amount of the right product to the right place at the right time and usually refers to supply chain problems (Tilanus, 1997: 7-16). In other words, logistics deals with the planning and the control of material flows and related information in organizations, both in the public and private sectors (Ghiani et al, 2004: 1)

The delivery of goods from plants to warehouses and from warehouses to local customers at minimum costs is an important and practical problem of a logistics manager. In many sectors of the economy, transportation costs amount to nearly a fifth or even a quarter of the average sales (Schneider, 1985: 118-126). Transportation adds value to the economic activities and should be deal with great care to provide savings.

The cost of transport is the payment for shipment between two geographical locations (Bowersox et al., 2002: 41) If the minimum transportation cost of a distribution system which has multi plants, multi warehouses and multi retailers wanted to be found, linear programming model can be used with objective function and capacity constraints. This model called as a transportation model.

Transportation model which was first studied by the Russian mathematician, L.V. Kantorovich (O’Neill and Kelly, 1991) is an application of network flow problem and it is a special case of the linear programming problem with one flow balance constraint for each node and two capacity constraints for each arc. Network flow problems are most widely applied to real systems (Calvete, 2003: 585-600).

A large number of problems in transportation, communications and manufacturing problems can be modeled as linear programming or network flow problems. Because of the model flexibility and very efficient algorithms have been developed for such problems.

In a distribution system if you have too many variables, the transportation model can not be solved easily.

In this paper, a multi-warehouse multi-retailer distribution system, considered from the real life, has been modeled as a network flow problem or a linear programming problem. The model has 2 plants, 11 warehouses and 100 retailers (113 nodes, 1122 arcs). Genetic Algorithm (GA), which is a heuristic optimization technique has been used to find the optimum solution (minimum cost) of the model. In the end the results are reported and compared with the results which are calculated with Excel premium solver and Lingo 4.0.

After this introduction, in Section 2, the mathematical notation of the problem appears. In section 3, we introduce the Genetic Algorithm and its steps. Then we propose the algorithm to find all efficient solutions and give an example that will help to understand the algorithm. In this section we also expose the computer results of our method. In the last section, we will conclude with some comments.
2. MATHEMATICAL MODEL OF THE PROBLEM

The objective of a transportation model is to determine how many units to ship along each road so as to minimize the total shipping costs, while satisfying the source and retailer requirements. The quantity shipped from node $i$ to node $j$, called the flow on arc $(i, j)$, is represented by $x_{ij}$ (Sun, 2002: 629-647).

Consider a network flow problem as Fig. 1, the circles represent plants, warehouses and retailers (called nodes) and the lines are paths between nodes (called arcs). Let $G = (N, A)$ be a directed network consisting of a finite set of nodes, $N$, and a set of directed arcs, $A$, linking pairs of nodes in $N$. We associate with every arc of $(i, j) \in A$, a flow $x_{ij}$, a cost per unit flow $c_{ij}$, a lower bound of the flow $l_{ij}$ and an upper bound or capacity $u_{ij}$ (Sedeno-Noda and Gonzalez-Martin, 2001: 139-156).
\( (i, j) \in A \), as before, let \( l_{ij} \) and \( u_{ij} \) denote, respectively, the lower and upper bounds on flow on arc \( (i, j) \) (Ahuja at all, 2002: 141-148) (Noda and Martin, 2001: 139-147).

3. GENETIC ALGORITHM

Genetic algorithms are search and optimization methods which were developed initially by J. Holland and his associates at the University of Michigan in the 1960s and 1970s (Reeves, 1995: 152 and Bertel and Billaut, 2004: 656). Genetic Algorithm maintains a population of solution candidates and evaluates the quality of each solution candidate according to a problem-specific fitness function, which defines the environment for the evolution. New solution candidates are created by selecting relatively fit members of the population and recombining them through various operators (Allen and Karjalainen, 1999: 245-271).

In our problems, fitness function evaluates the total transportation cost. The supply and demand constraints are considered penalty functions in fitness functions.

In our problems, chromosomes have been determined in two ways according to divisibility of the order quantity. If the division of the order is allowed, each gene of chromosomes represent the quantity of the products, flowing from a plant 1 to a warehouse 1 and from warehouse 1 to retailer 1. If the division of the order is not allowed, the first part of the double substring of the chromosomes represents the plant which supplies related retailer and the second part of it represents the warehouse which supplies related retailer (Plant-Warehouse Encoding, P-W Encoding).

![Chromosome structure belonging to divisible order](image)

![Chromosome structure belonging to indivisible order](image)
demand 1 + demand 3 + \ldots + demand (n+1)/2 = total product quantity flowing from plant 1

demand 2 + \ldots + demand (n+1)/2 = total product quantity flowing through warehouse 5

3.1. Crossover

Crossover along with selection reproduction and mutation is one of the main operators of any GA. It provides an exchange of design characteristics between paired individuals (designs) (Hasançebi and Erbatur, 2000: p.435). According to the size of the problem single-point, two-point or multi-point crossover are used.

![Two-point crossover diagram](image)

3.2. Mutation

Mutation plays decidedly secondary role in the operation of GA. In artificial genetic systems the mutation operator protects against such an irrecoverable loss (Goldberg, 1989: p.14).

Mutation rate is usually small like 0.001. 0.4-0.6 mutation rate has been used because of the structure of the problem in this paper. This high mutation rate causes the mutated supply balance.

3.3. Balance Operator

The mutated supply balance after mutation operator is arranged by using balance operator. The balance operator changes plant and warehouse which the retailers demand from so that the searching of global minimum is being faster.

3.4. The Steps of Overall Procedure

The overall procedure for solving the problem as follows (Mattfeld and Bierwirth, 2004: 616-630)
3.5. Numerical Example and The Results

We implement our proposed method using Visual Basic for Applications. The performance of the proposed method is tested by using for different problems as given in Table 1.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Number of plants</th>
<th>Number of warehouses</th>
<th>Number of retailers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>11</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>11</td>
<td>100</td>
</tr>
</tbody>
</table>

We summarize the results of all of the problems in the following Table 2.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Population size</th>
<th>Maximum generation</th>
<th>Computational time (in minutes)</th>
<th>GA results</th>
<th>Excel Premium-Solver results</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>1,35</td>
<td>1</td>
<td>1.310</td>
<td>1.310</td>
</tr>
<tr>
<td>2</td>
<td>40</td>
<td>1.000</td>
<td>0,8</td>
<td>1.310</td>
<td>1.310</td>
</tr>
<tr>
<td>3</td>
<td>50</td>
<td>15.000</td>
<td>23,6</td>
<td>3.343.450</td>
<td>3.343.400</td>
</tr>
<tr>
<td>4</td>
<td>200</td>
<td>35.000</td>
<td>74</td>
<td>2.368.750</td>
<td>2.341.130</td>
</tr>
<tr>
<td>5</td>
<td>300</td>
<td>50.000</td>
<td>135</td>
<td>2.333.850</td>
<td>2.323.740</td>
</tr>
</tbody>
</table>

Stopping criterion met? [Yes, No]

Final solution
The plants and the warehouses which supply first 20 retailers are shown in Table 3 for Problem 2.

### Table 3. The results of the problem 2

<table>
<thead>
<tr>
<th>Plant</th>
<th>Warehouse</th>
<th>Retailer</th>
<th>Demand</th>
<th>Plant</th>
<th>Warehouse</th>
<th>Retailer</th>
<th>Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>60</td>
<td>1</td>
<td>1</td>
<td>11</td>
<td>20</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
<td>2</td>
<td>60</td>
<td>1</td>
<td>1</td>
<td>12</td>
<td>30</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>3</td>
<td>80</td>
<td>1</td>
<td>7</td>
<td>13</td>
<td>40</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>4</td>
<td>40</td>
<td>1</td>
<td>1</td>
<td>14</td>
<td>30</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>5</td>
<td>40</td>
<td>2</td>
<td>1</td>
<td>15</td>
<td>30</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>6</td>
<td>100</td>
<td>2</td>
<td>5</td>
<td>16</td>
<td>110</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>7</td>
<td>90</td>
<td>2</td>
<td>1</td>
<td>17</td>
<td>50</td>
</tr>
<tr>
<td>1</td>
<td>8</td>
<td>8</td>
<td>20</td>
<td>1</td>
<td>1</td>
<td>18</td>
<td>30</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>9</td>
<td>100</td>
<td>2</td>
<td>1</td>
<td>19</td>
<td>60</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>10</td>
<td>110</td>
<td>2</td>
<td>5</td>
<td>20</td>
<td>90</td>
</tr>
</tbody>
</table>

4. CONCLUSION

In this paper we proposed a GA approach to find the best network flow plan in transshipment system. It has been seen from the results that for the small size problems, this method can search the optimal solution in all of the time. But in the big size network flow problems, the optimal solution can not be got easily in a short time.

We utilize the P-W encoding to find the solution of big size network flow problems. In our last problem, normally there are 44 alternatives (4 plant * 11 warehouses) for each retailer. The number of variables is reduced from 4400 to 200 and the solution process make faster by using P-W encoding. In P-W encoding, there exists two variables for each retailer. First of these two variables is the plant which related retailers demand from, and the second one represents the warehouse that retailers demand.

We try to find the minimum cost of the transshipments and we have not considered the fixed costs of the plants and warehouses. One may can add fixed costs of the warehouses to this model.

We suggest that this algorithm can be more efficient method for multi-stage network problems if the algorithm can be more developed.
REFERENCES


